Odd Cycle Games and Connected Rules

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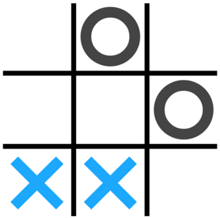

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For what values of b do Breaker and Waiter win?

The point where the winner switches is referred to as the **bias threshold**, denoted by b_{mb} and b_{cw}.

Examples of Positional Games

Let the board X be given by all edges of the complete graph on n vertices.

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The winning sets of the **connectivity game** consist of all spanning trees of K_n. Gebauer and Szabó showed that $b_{\mathrm{mb}} \approx n/\ln n$.

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The winning sets of the **cycle game** are all cycles in K_n. In the **odd (even) cycle game** the winning sets are all odd (even) cycles.

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Theorem 1 (Corsten, Mond, Pokrovskiy, S. and Szabó ′19+)

In the Maker-Breaker odd cycle game

$$b_{mb} \geq \left(\frac{4 - \sqrt{6}}{5} - o(1) \right) n \approx 0.3101n.$$

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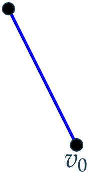

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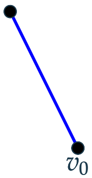

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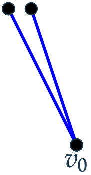

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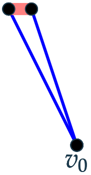

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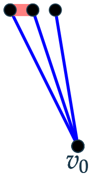

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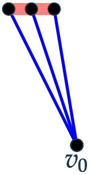

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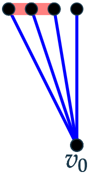

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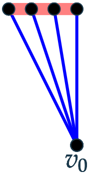

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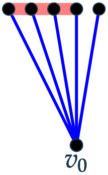

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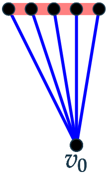

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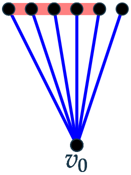

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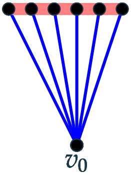

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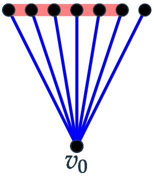

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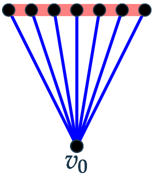

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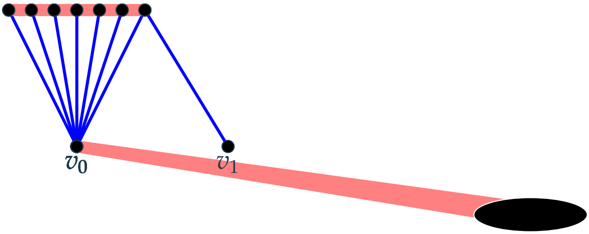

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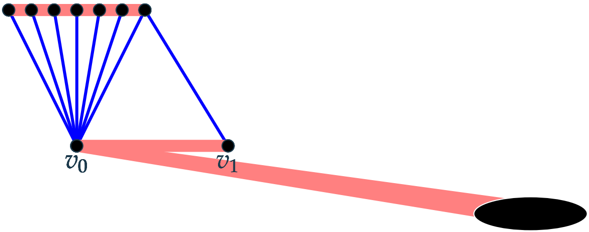

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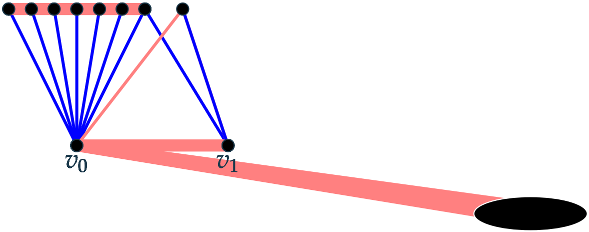

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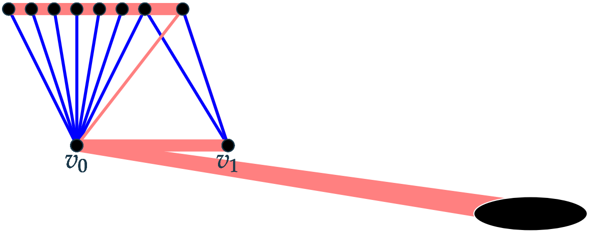

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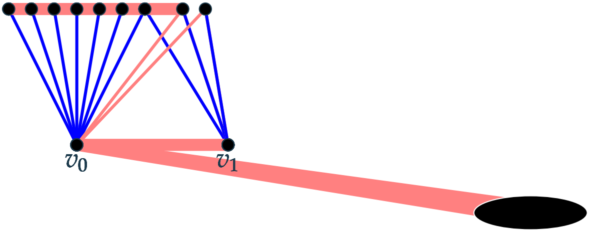

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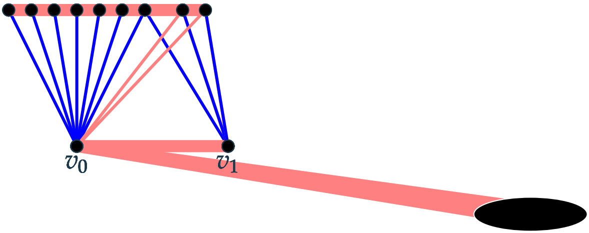

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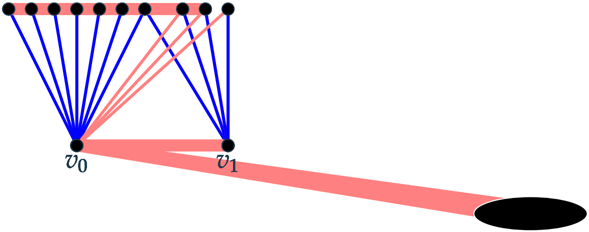

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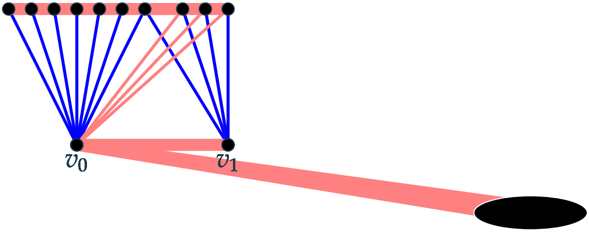

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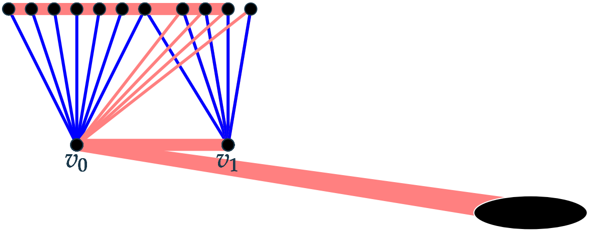

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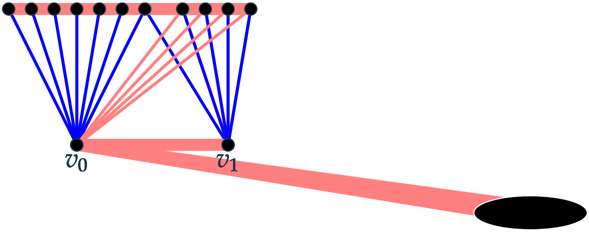

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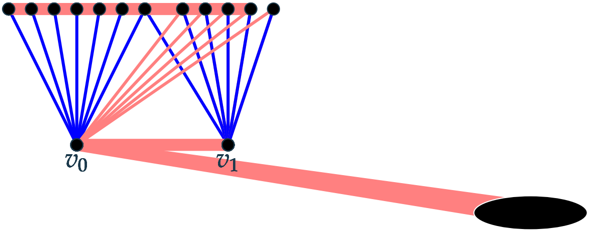

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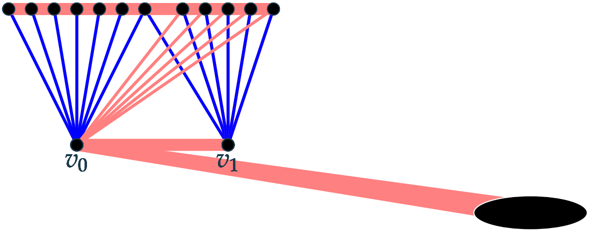

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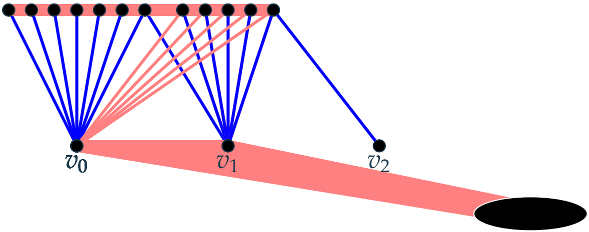

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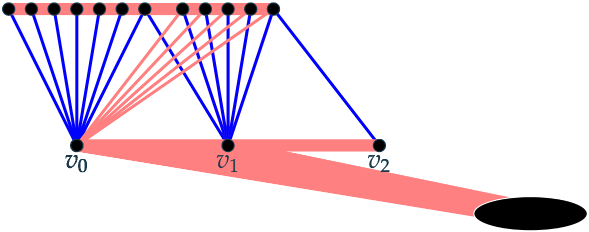

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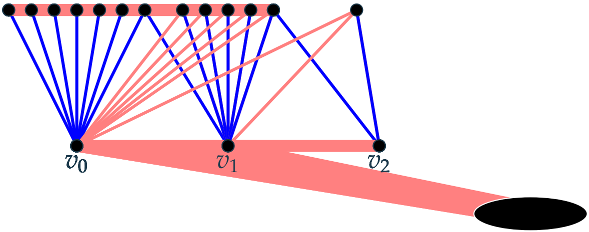

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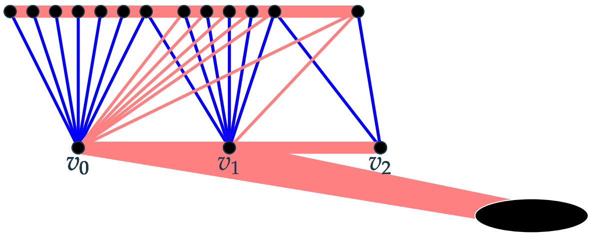

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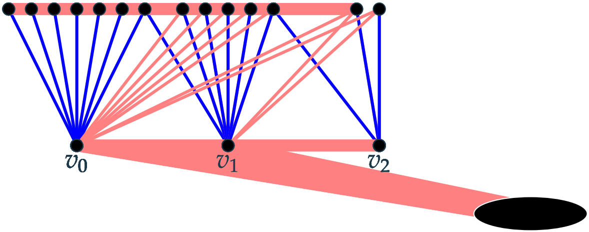

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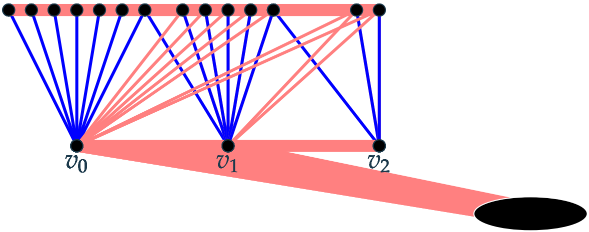

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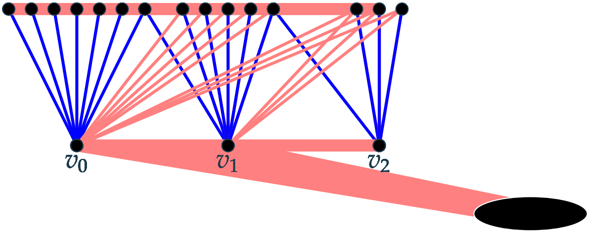

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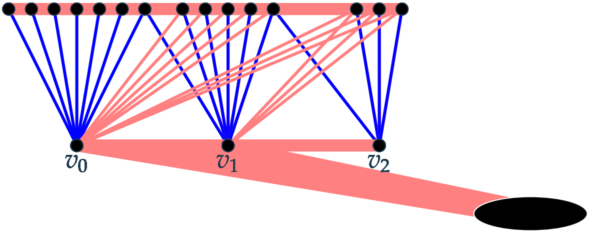

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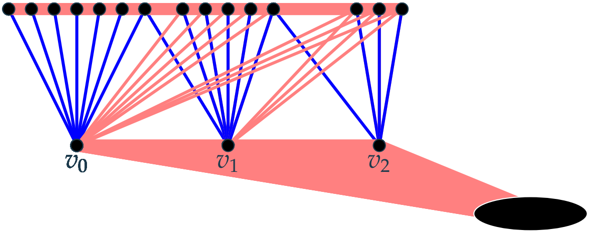

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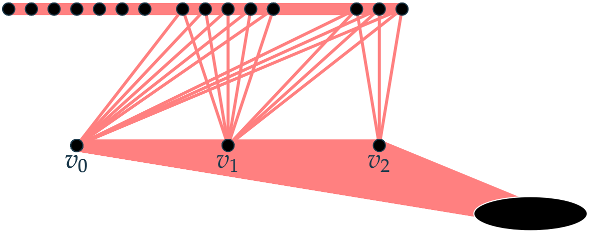

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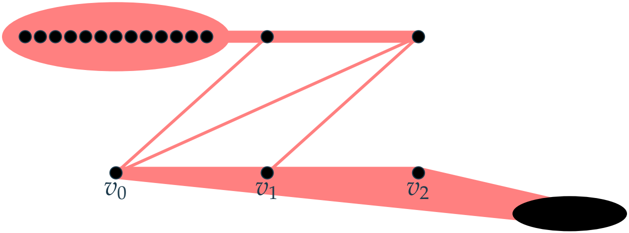

Connected Maker-Breaker Cycle Games

Theorem (Bednarska and Pikhurko 2008)

In the Maker-Breaker odd cycle game $b_{mb} \geq 0.2928n$.

Theorem 1 (Corsten, Mond, Pokrovskiy, S. and Szabó ′19+)

In the Maker-Breaker odd cycle game $b_{mb} \geq 0.3101n$.

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A Strategy for Breaker under Connected Rules

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3. This way Breaker minimises the number of edges ending up between the two parts of Maker's graph.

Client-Waiter Cycle Games

Theorem (Hefetz, Krivelevich, and Tan 2016)

In the Client-Waiter cycle game $b_{cw} = \lceil n/2 \rceil - 1$.

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We have $b_{cw} = n/2 - o(n)$ *in the odd cycle Client-Waiter game.*

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3. Whenever offering an edge incident to a vertex not yet in Client's graph, Waiter must either *offer all unclaimed edges between that vertex and Client's graph*

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4. Client tries to reduce the number of times the later occurs.

Open Question

Q1. What is the threshold bias for other variants of the odd cycle games, for example Avoider-Enforcer or Waiter-Client?

Q2. What is the threshold bias for the connected Maker-Breaker H-game?

Q3. One can view the odd cycle game as the non-2-colourability game. It was proved by Hefetz et al. that the threshold bias for the Maker-Breaker non-k-colourability game satisfies $b_{mb} = \Theta_k(n)$. Do we have $b_{mb} \approx b^c_{mb}$?

Thank you for your attention!